ON THE SPECIAL THEORY OF RELATIVITY AND THE LORENTZ FACTOR

By Relatively Special

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Why We Want To Go

Last year was the hundred year anniversary of the publication of the general theory of relativity by Albert Einstein and also the year that LIGO first measured gravitational waves, making a huge impact not only in the scientific community, but additionally in news all over the world. This turning point helped inspire us to delve deeper into Einstein's theories and their implications in the natural world and made us feel that it is quite appropriate to base an experiment on one of his many great discoveries. Further research led us to his ground breaking work on special relativity in which our whole team took great interest.

This competition gives us the invaluable opportunity to broaden our knowledge in this field by conducting an experiment with the guidance of the experts at CERN. We would be able to plan and carry out our experiment with fewer limitations and on a far grander scale compared to a school environment, which is why this chance means so much to us.

Our experiment

Aim

Our experiment aims to test the validity of the Lorentz factor by measuring the effect of time dilation due to special relativity on the decay rate of pions.

Theory

The special theory of relativity states that the speed of light is the same in all reference frames. This leads to changes in properties of an object as it moves faster in the reference frame of an observer. These changes can be in mass, length and time experienced.

The factor by which these changes occur is given by the Lorentz factor, γ:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

To calculate relativistic time dilation, the following equation involving the Lorentz factor is utilised:

$$\Delta t = \gamma \Delta t_0$$

Where t_0 is the time experienced by the observer.

If an unstable particle increases its velocity with respect to an observer, the Lorentz factor will increase resulting in it travelling through time at a slower rate. This means that the mean lifetime will be perceived to be greater than when it is at rest. Similarly, the fast moving particle's mass will increase in the reference frame of the observer resulting in a decrease in velocity due to conservation of momentum. This means that relativistic effects must be taken into account when dealing with particles travelling at high velocities.

Method 1: Momentum

Relativistic mass is given by:

$$m = \gamma m_0$$

Therefore relativistic momentum:

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This equation can be rearranged to give velocity:

$$v = \pm \frac{pc}{\sqrt{{m_0}^2 c^2 + p^2}}$$

The momentum given to the particles in the accelerator, p, the speed of light in a vacuum, c, and the rest mass of the pions, m_0 ($140 MeV/c^2$), is known. This means that the theoretical velocity can be calculated using this equation.

Method 2: Decay

The proportion of particles that have not decayed after a time, t, is given by:

$$P_t = e^{-\lambda t}$$

The decay rate, λ, is the reciprocal of the mean lifetime, μ ($2.6 \times 10^{-8} s$), and relativistic time dilation of the mean lifetime is:

$$\mu = \gamma \mu_0$$

Therefore the proportion that remain:

$$P_t = e^{-\frac{t}{\gamma \mu_0}}$$

This equation can be rearranged to give velocity:

$$v = \pm c\sqrt{1 - \left(\frac{\mu_0 \ln(P_t)}{t}\right)^2}$$

The time that the particles take to travel between two Cherenkov counters, t, can be measured and the proportion of particles remaining calculated:

$$P_t = \frac{pions\ in\ final\ Cherenkov}{pions\ in\ first\ Cherenkov}$$

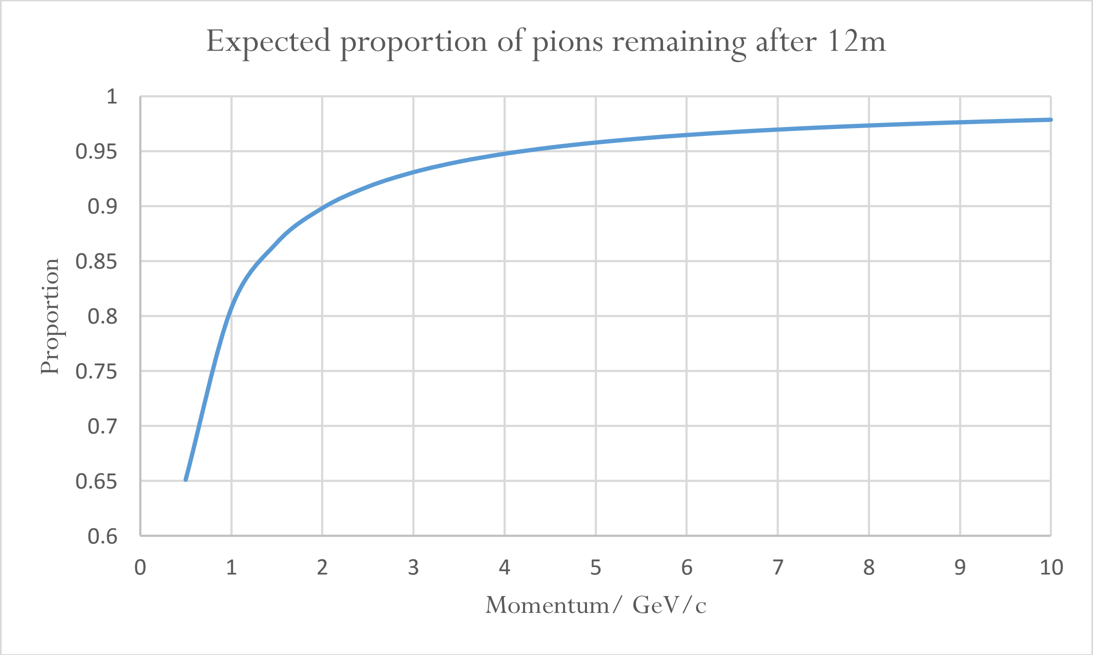
This means that the theoretical velocity can also be calculated using this equation.

Summary

The actual velocity can be found by measuring the time taken for the pions to travel between two scintillators which are a known distance apart.

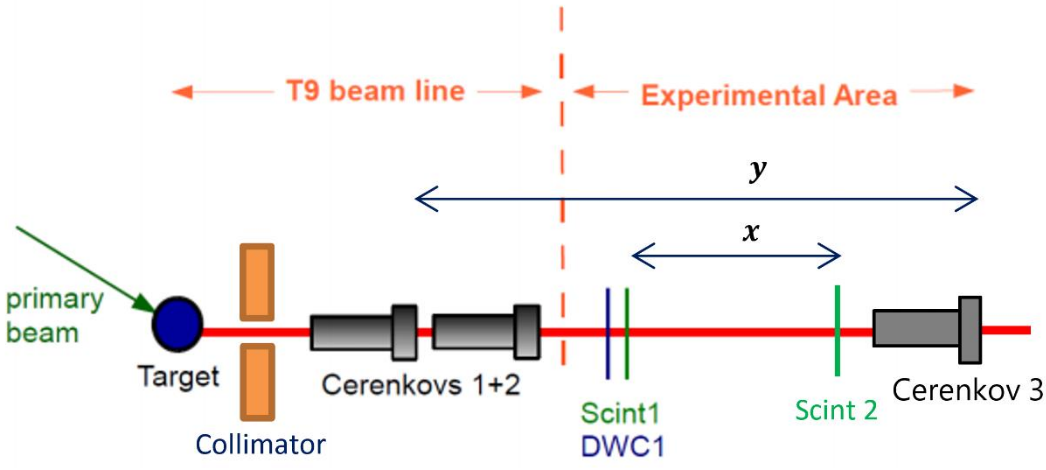
If the Lorentz factor is valid, the two calculated velocities and the actual velocity are expected to be equal (within experimental accuracy):

$$v = \frac{pc}{\sqrt{{m_0}^2c^2 + p^2}} = c\sqrt{1 - \left(\frac{\mu_0 \ln(P_t)}{t}\right)^2}$$



This graph shows our theoretical predictions for the number of decaying particles having travelled 12 metres through the experimental area. It indicates that there should be a measurable difference in the number of particles at different momenta.

Experimental set up



Method

We decided upon using a positive beam as this will contain a larger abundance of pions but a negative beam could also be used with a greater percentage in comparison to other particles.

The primary beam will collide with the target and produce an array of particles including pions. The use of the horizontal collimator will allow the removal of any particles with a lower momentum than specified and thus leave the beam with a more precise figure. Absorbers have not been used on the beam's path as the momentum must be kept constant to make our measurements more accurate. The second Cherenkov counter will be emptied but the first will be filled. This will be used to identify and count the number of pions contained in the beam initially. The first scintillator will count the number of particles entering and a second scintillator has been placed in the set up at a distance, x, away from the first one. The time taken for the pions to travel between these points can be measured and the velocity of the beam calculated. The third Cherenkov counter will count the number of pions left in the beam after they have travelled the distance, y, as some will spontaneously decay into other products such as muons and electrons. The number that are still left will determine how many have decayed.

What we hope to take away

If we find ourselves in the privileged position to have conducted our experiment at CERN, we hope not to be the only people benefiting from the experience. We would like to promote physics in our school to raise interest amongst our fellow students by sharing our adventure about what it is like to go into research. We believe this sort of motivation is vital for the successive generations of young scientists in order to help them learn the tools to make the next scientific breakthroughs and propel the human race to places previously out of reach.

Acknowledgements

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